Foundation Mathematics for Biosciences

Ela Bryson & Jackie Willis

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Foundation Mathematics for Biosciences

First Edition

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Dedication

To my wonderful Mum,
Ela

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Contents

Preface

Whether you have already purchased this book or are still contemplating buying it, we hope you will take some time reading this preface so that you can understand why this book was written and how to get the most out of it.

The Purpose of this Book

The authors have spent many years supporting students with the mathematical demands of undergraduate and postgraduate courses in the biosciences. We believe that you will benefit from our experience and the immense effort that we have poured into this book so that you become successful in both your degree course and future career.

Content

This book consists of twelve chapters and each chapter is divided into two sections. It is designed to allow you progress in a logical manner from sets of easier, fundamental problems to much more demanding and complex calculations aligned to various disciplines in biology.

In the first five chapters we cover the essential ground rules to enable a smooth transition into the later chapters. We begin with the arithmetic operations in mathematics in Chapter 1, giving emphasis to the use of equations and indices. In Chapter 2 we move on to fractions and here you will also learn about the rounding of numbers and scientific notation. Chapter 3 introduces the SI units of measurement and rules for their use and conversions between different units. Ratios and percentages are discussed in Chapter 4, providing examples of calculations encountered when preparing mixtures and solutions with a given percentage concentration. Chapter 5 is dedicated to logarithms, giving clear explanations of the laws of logarithms and the application of logarithms in the biosciences.

In Chapter 6 you will learn about preparing molar solutions and both standard and serial dilutions. We know this is a problem area for many students, hence our decision to devote a whole chapter to these topics.

Chapters 7-10 present calculations relevant to the specialisms in biosciences. Each chapter provides a brief overview of some of the theoretical concepts of each topic before working through typical calculations. Chapter 7 covers measurements made in microscopy, cell biology and microbiology as well as calculations of selected physiological and pharmacological parameters. Chapter 8 focuses on calculations relating to a range of techniques used in analytical biology and radiobiology. Chapter 9 contains examples of solutions to problems in DNA and protein analysis, whilst Chapter 10 is devoted to enzyme kinetics, including analysis of enzyme inhibition.

In Chapter 11 you are introduced to statistics and will conduct some statistical analysis. Chapter 12 demonstrates how to present data correctly in graphs and charts as well as explore relationships between variables using correlation and regression analysis.

Key Features

- **Learning Outcomes**
  A summary is provided at the start of each chapter of the learning outcomes expected to be achieved once the chapter has been completed. This will help you keep track of what you have learnt.

- **Worked Examples**
  Throughout the book there are numerous worked examples with detailed solutions and explanations, taking you step by step through each calculation.

- **SELF-ASSESSMENT**
  There are also calculations for you to attempt independently, then check against the answer key at the end of the book. This will help you check your understanding and increase confidence as problems become progressively more difficult.

- **MyMathLabGlobal**
  This book is available with access to the online resource, MyMathLabGlobal, but requires that a course ID has been set up by your tutor for you to use it. This e-resource provides an extensive bank of exercises developed by the authors to provide the opportunity for further self-assessment (examples of these questions are listed at the end of each half chapter of the book). MyMathLabGlobal will guide you through each step in solving a problem until the fully worked correct answer is displayed. Your tutor has the option to set up homework, quizzes and tests.

- **Key Terms**
  Key terms are defined in each chapter and these are highlighted in **coloured text** where they are explained. A list of key terms is also given at the end of the chapter, indicating those which may appear as a key term in other chapters of the book. Reviewing the key terms once a chapter is completed will ensure you fully understand each concept and are ready to progress further.

  In the event that Pearson invite us to produce a second edition, we would like to hear your suggestions on any improvements or additional material that could be included. We can be contacted at: mathsforbiosciences@gmail.com.

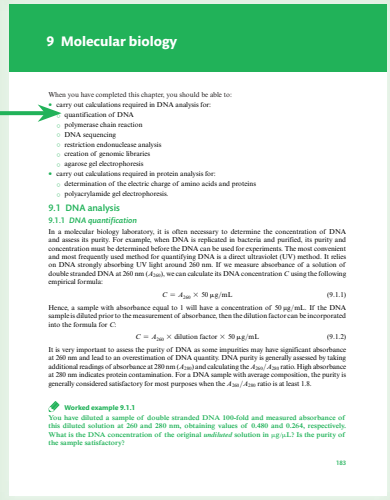
Thank you for purchasing this book, we hope you will enjoy using it.

Ela Bryson
Jackie Willis

Guided tour

Learning outcomes

Learning outcomes are listed at the start of each chapter to show what you can learn.



9 Molecular biology

When you have completed this chapter, you should be able to:

- carry out calculations required in DNA analysis for:
  - quantification of DNA
  - polymerase chain reaction
  - DNA sequencing
  - restriction endonuclease analysis
  - creation of genomic libraries
  - agarose gel electrophoresis
- carry out calculations required in protein analysis for:
  - determination of the electric charge of amino acids and proteins
  - polyacrylamide gel electrophoresis.

9.1 DNA analysis

9.1.1 *DNA quantification*

In a molecular biology laboratory, it is often necessary to determine the concentration of DNA and assess its purity. For example, when DNA is replicated in bacteria and purified, its purity and concentration must be determined before the DNA can be used for experiments. The most convenient and most frequently used method for quantifying DNA is a direct ultraviolet (UV) method. It relies on DNA strongly absorbing UV light around 260 nm. If we measure absorbance of a solution of double stranded DNA at 260 nm (A_{260}), we can calculate its DNA concentration C using the following empirical formula:

$$C = A_{260} \times 50\ \mu g/mL \qquad (9.1.1)$$

Hence, a sample with absorbance equal to 1 will have a concentration of 50 μg/mL. If the DNA sample is diluted prior to the measurement of absorbance, then the dilution factor can be incorporated into the formula for C:

$$C = A_{260} \times \text{dilution factor} \times 50\ \mu g/mL \qquad (9.1.2)$$

It is very important to assess the purity of DNA as some impurities may have significant absorbance at 260 nm and lead to an overestimation of DNA quantity. DNA purity is generally assessed by taking additional readings of absorbance at 280 nm (A_{280}) and calculating the A_{260}/A_{280} ratio. High absorbance at 280 nm indicates protein contamination. For a DNA sample with average composition, the purity is generally considered satisfactory for most purposes when the A_{260}/A_{280} ratio is at least 1.8.

Worked example 9.1.1

You have diluted a sample of double stranded DNA 100-fold and measured absorbance of this diluted solution at 260 and 280 nm, obtaining values of 0.480 and 0.264, respectively. What is the DNA concentration of the original *undiluted* solution in μg/μL? Is the purity of the sample satisfactory?

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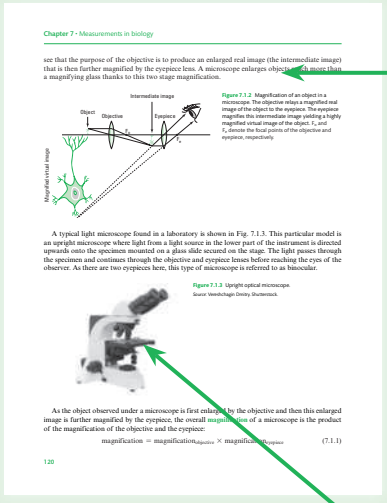
Theoretical background

The text and illustrations explain underpinning theoretical concepts as well as reflecting the practical nature of the biosciences.



Chapter 7 - Measurements in biology

see that the purpose of the objective is to produce an enlarged real image (the intermediate image) that is then further magnified by the eyepiece lens. A microscope enlarges objects much more than a magnifying glass thanks to this two stage magnification.

Figure 7.1.2 Magnification of an object in a microscope. The objective relays a magnified real image of the object to the eyepiece. The eyepiece magnifies this intermediate image yielding a highly magnified virtual image of the object. F_o and F_e denote the focal points of the objective and eyepiece, respectively

A typical light microscope found in a laboratory is shown in Fig. 7.1.3. This particular model is an upright microscope where light from a light source in the lower part of the instrument is directed upwards onto the specimen mounted on a glass slide secured on the stage. The light passes through the specimen and continues through the objective and eyepiece lenses before reaching the eyes of the observer. As there are two eyepieces here, this type of microscope is referred to as binocular.

Figure 7.1.3 Upright optical microscope.
Source: Veselchagin Dmitry, Shutterstock.

As the object observed under a microscope is first enlarged by the objective and then this enlarged image is further magnified by the eyepiece, the overall magnification of a microscope is the product of the magnification of the objective and the eyepiece:

$$\text{magnification} = \text{magnification}_{\text{objective}} \times \text{magnification}_{\text{eyepiece}} \qquad (7.1.1)$$

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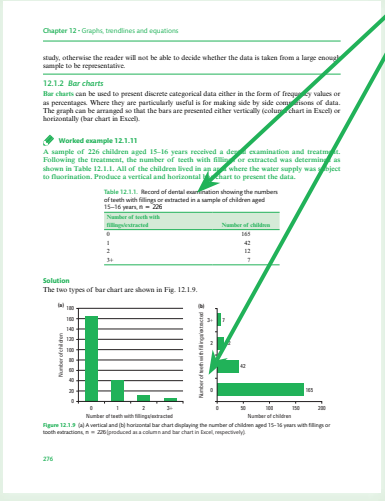
Figures

Graphs, tables, diagrams and photographs are included to illustrate examples.



Chapter 12 - Graphs, trendlines and equations

study, otherwise the reader will not be able to decide whether the data is taken from a large enough sample to be representative.

12.1.2 *Bar charts*

Bar charts can be used to present discrete categorical data either in the form of frequency values or as percentages. Where they are particularly useful is for making side by side comparisons of data. The graph can be arranged so that the bars are presented either vertically (column chart in Excel) or horizontally (bar chart in Excel).

Worked example 12.1.11

A sample of 226 children aged 15–16 years received a dental examination and treatment. Following the treatment, the number of teeth with fillings or extracted was determined as shown in Table 12.1.1. All of the children lived in an area where the water supply was subject to fluorination. Produce a vertical and horizontal bar chart to present the data.

Table 12.1.1. Record of dental examination showing the numbers of teeth with fillings or extracted in a sample of children aged 15–16 years, n = 226

Number of teeth with fillings/extracted	Number of children
0	165
1	42
2	12
3+	7

Solution

The two types of bar chart are shown in Fig. 12.1.9.

Figure 12.1.9 (a) A vertical and (b) horizontal bar chart displaying the number of children aged 15–16 years with fillings or tooth extractions, n = 226 (produced as a column and bar chart in Excel, respectively).

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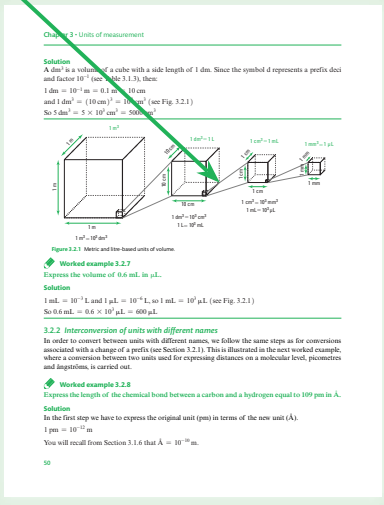

Chapter 3 - Units of measurement

Solution

A dm^3 is a volume of a cube with a side length of 1 dm. Since the symbol d represents a prefix deci and factor 10^{-1} (see Table 3.1.3), then:

1 dm = 10^{-1} m = 0.1 m = 10 cm

and 1 dm^3 = $(10\ cm)^3$ = 1000 cm^3 (see Fig. 3.2.1)

So 5 dm^3 = 5 × 10^3 cm^3 = 5000 cm^3

Figure 3.2.1 Metric and litre-based units of volume.

Worked example 3.2.7

Express the volume of 0.6 mL in μL.

Solution

1 mL = 10^{-3} L and 1 μL = 10^{-6} L, so 1 mL = 10^3 μL (see Fig. 3.2.1)

So 0.6 mL = 0.6 × 10^3 μL = 600 μL

3.2.2 *Interconversion of units with different names*

In order to convert between units with different names, we follow the same steps as for conversions associated with a change of a prefix (see Section 3.2.1). This is illustrated in the next worked example, where a conversion between two units used for expressing distances on a molecular level, picometres and ångströms, is carried out.

Worked example 3.2.8

Express the length of the chemical bond between a carbon and a hydrogen equal to 109 pm in Å.

Solution

In the first step we have to express the original unit (pm) in terms of the new unit (Å).

1 pm = 10^{-12} m

You will recall from Section 3.1.6 that Å = 10^{-10} m.

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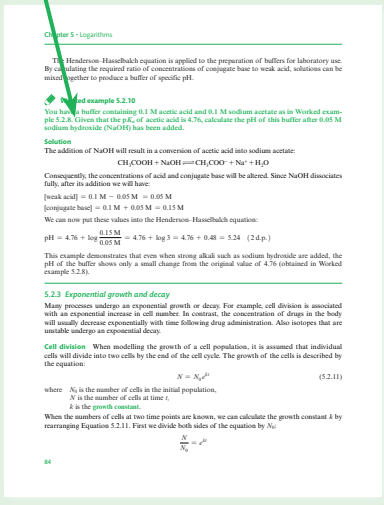

Chapter 5 - Logarithms

The Henderson–Hasselbalch equation is applied to the preparation of buffers for laboratory use. By calculating the required ratio of concentrations of conjugate base to weak acid, solutions can be mixed together to produce a buffer of specific pH.

Worked example 5.2.10

You have a buffer containing 0.1 M acetic acid and 0.1 M sodium acetate as in Worked example 5.2.8. Given that the pK_a of acetic acid is 4.76, calculate the pH of this buffer after 0.05 M sodium hydroxide (NaOH) has been added.

Solution

The addition of NaOH will result in a conversion of acetic acid into sodium acetate:

$$CH_3COOH + NaOH \rightleftharpoons CH_3COO^- + Na^+ + H_2O$$

Consequently, the concentrations of acid and conjugate base will be altered. Since NaOH dissociates fully, after its addition we will have:

[weak acid] = 0.1 M − 0.05 M = 0.05 M

[conjugate base] = 0.1 M + 0.05 M = 0.15 M

We can now put these values into the Henderson–Hasselbalch equation:

$$pH = 4.76 + \log\frac{0.15\ M}{0.05\ M} = 4.76 + \log 3 = 4.76 + 0.48 = 5.24 \quad (2\ d.p.)$$

This example demonstrates that even when strong alkali such as sodium hydroxide are added, the pH of the buffer shows only a small change from the original value of 4.76 (obtained in Worked example 5.2.8).

5.2.3 *Exponential growth and decay*

Many processes undergo an exponential growth or decay. For example, cell division is associated with an exponential increase in cell number. In contrast, the concentration of drugs in the body will usually decrease exponentially with time following drug administration. Also isotopes that are unstable undergo an exponential decay.

Cell division When modelling the growth of a cell population, it is assumed that individual cells will divide into two cells by the end of the cell cycle. The growth of the cells is described by the equation:

$$N = N_0 e^{kt} \qquad (5.2.11)$$

where N_0 is the number of cells in the initial population,
N is the number of cells at time t,
k is the growth constant.

When the numbers of cells at two time points are known, we can calculate the growth constant k by rearranging Equation 5.2.11. First we divide both sides of the equation by N_0:

$$\frac{N}{N_0} = e^{kt}$$

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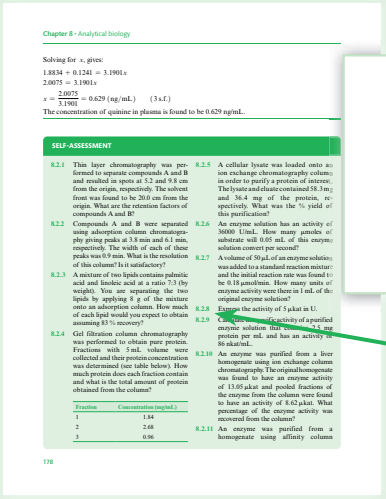
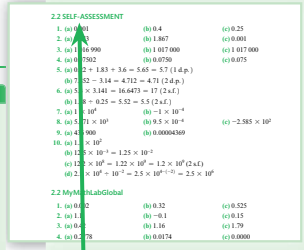
Worked examples

Worked examples are provided throughout, with clear step-by-step explanations to guide you through each problem.





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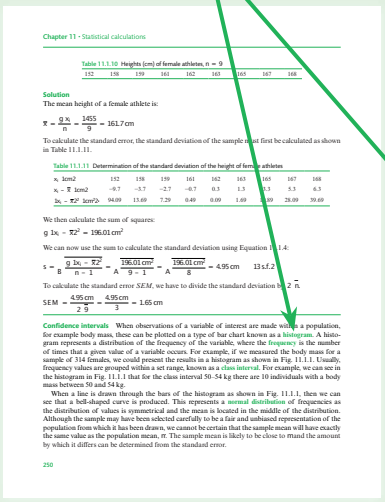
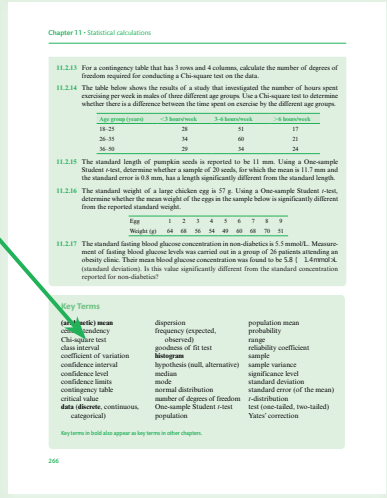
Key terms
Key terms are defined and clearly highlighted in the text. To aid revision, there is a list at the end of each chapter.





About the authors

Ela Bryson received a Master's degree in Molecular Biology from the University of Lodz in Poland, an MSc in Physics from the University of York and a PhD in Biophysics from The Open University. Her postdoctoral research focused on protein folding and Huntington disease. Ela is currently a Senior Lecturer at the University of Hertfordshire where she has been teaching molecular biology as well as mathematics and statistics to Biosciences and Pharmacy students.

Jackie Willis was awarded a BSc in Biochemistry and a PhD in Clinical Pharmacology and Therapeutics by Birmingham University. Jackie has taught molecular pharmacology, mathematics and statistics at Coventry University and the University of Hertfordshire and has previously published a textbook on statistics for Biosciences undergraduates. Jackie retired as an Associate Dean in 2015 having spent more than 30 years working in academia.

Both authors were presented with a joint award by the University of Hertfordshire in recognition of their commitment to teaching mathematics.

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Ela Bryson and Jackie Willis

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1 Basic arithmetic skills

When you have completed this chapter, you should be able to:

- solve mathematical problems using appropriate operations and apply the principles of BODMAS
- rearrange and solve equations
- apply the laws of indices
- carry out calculations using a scientific calculator and provide your own estimate of the answer.

1.1 Elementary arithmetic calculations in biology

1.1.1 *Introduction*

Whether working in the laboratory or out in the field, biologists need to use elementary arithmetic skills. In this section, we will consider the fundamental rules of arithmetic and then apply these to some examples of basic calculations in biology. You should already be familiar with the operations used in this chapter, but you may find it useful to refer to Appendix 1 which provides a brief description of arithmetic operations and their symbols.

The information collected during biological investigations consists of a series of observations, referred to as the **data** (plural), where each individual observation is the **datum** (singular). Although data may be **qualitative** (such as the *colour* of fungal colonies growing on media), more frequently data are **quantitative** (such as the *number* of fungal colonies counted on a plate). Quantitative data can be expressed as fractions (e.g. $\frac{1}{2}$), percentages (50 %) or decimal numbers (0.5). However, some data are in the form of whole numbers. Whole numbers are referred to as **integers**. Integers are described as **discrete data** because they are whole numbers that have other numbers lying in between them. In this chapter we will be using integers.

1.1.2 *Basic operations*

Operations are the processes used to perform mathematical calculations. These include four basic operations: addition, subtraction, multiplication and division but there are many more (e.g. percentages, powers) that will be covered in later chapters. We will work through a couple of problems to remind you of how basic operations are used.

Worked example 1.1.1

In an investigation about the germination of cress seeds, a plant biologist wants to summarise the quantitative data collected about the germination of the seeds in a sample of 7 pots (Fig. 1.1.1).

Figure 1.1.1 Germinated cress seeds.

Five hundred seeds were sown in each pot and after 2 weeks the number of seeds that germinated was counted:

Pot 1	**326**
Pot 2	**402**
Pot 3	**397**
Pot 4	**420**
Pot 5	**381**
Pot 6	**368**
Pot 7	**352**

Solution

In order to calculate the total number of seeds that have germinated, we must add the number that germinated in each pot:

$326 + 402 + 397 + 420 + 381 + 368 + 352 = 2646$

In maths, the total is also referred to as the **sum** – the result of adding two or more numbers together. The investigator also needs to know how many seeds did not germinate in each pot. This can be calculated by subtracting the number that germinated from the number of seeds planted:

Pot 1	$500 - 326 = 174$
Pot 2	$500 - 402 = 98$
Pot 3	$500 - 397 = 103$
Pot 4	$500 - 420 = 80$
Pot 5	$500 - 381 = 119$
Pot 6	$500 - 368 = 132$
Pot 7	$500 - 352 = 148$

The number of seeds that failed to germinate is:

$174 + 98 + 103 + 80 + 119 + 132 + 148 = 854$

However, this is a very inefficient way of determining how many seeds did not germinate. As the same number of seeds was sown in each pot, by using multiplication we can easily calculate that the total number of seeds planted in seven pots was:

$500 \times 7 = 3500$

We can then calculate the number of seeds that did not germinate as the difference between the total number of seeds planted and the total number of seeds which germinated:

$3500 - 2646 = 854$

If the investigator wanted to express in general terms how many seeds per pot germinated, this can be determined by dividing the total number of seeds that germinated by the number of pots:

$$\frac{2646}{7} = 378$$

378 represents the **average**, or **arithmetic mean**, and these two terms are often used interchangeably. The arithmetic mean (usually just called the **mean**) represents the typical value within a set of numbers.

We can see that in Pot 4, 420 seeds germinated which is above average, whilst in Pot 1, 326 seeds germinated which is below average. In this example, the mean was calculated using the following general rule:

$$\text{mean} = \text{sum of observations in the set} \div \text{number of observations in the set}$$

We can think of this as being a word **equation** because it shows how to perform the calculation. In maths, we use symbols in equations to represent the operations used to process a calculation. In our word equation, the sum can be represented by the symbol $\sum$ (capital Greek letter sigma, meaning the sum of) and each observation by x_i. The number of observations is generally referred to as n and the symbol for the sample mean is $\overline{x}$. Using mathematical symbols, the word equation can be rewritten as:

$$\overline{x} = \frac{\sum x_i}{n}$$

You will learn more about the arithmetic mean in Section 11.1. Throughout this book there are equations which include symbols representing quantities and mathematical operations. As quantity symbols are generally single letters of the Latin or Greek alphabet, you may find it useful to familiarise yourself with other commonly used Greek letters which are listed in Appendix 1.

Worked example 1.1.2

In the laboratory, toxicological testing is frequently performed by exposing cells to a test substance to determine whether it causes the cells to die. As this testing is performed on a large scale, cellular suspensions are pipetted into small wells on a plate. These are known as multi-well plates, as shown in Fig. 1.1.2.

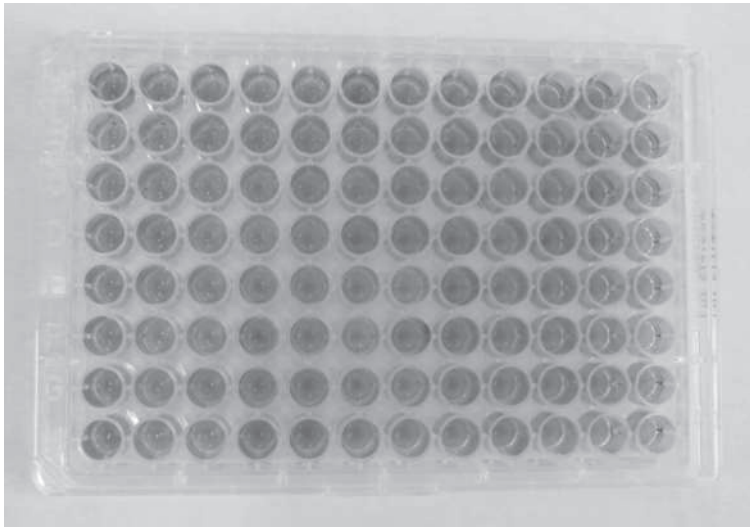

Figure 1.1.2 Multi-well plate.

How many wells are there on the plate? If the laboratory is contracted to perform 960 047 tests, how many multi-well plates will be required? Notice that the digits of the number 960 047 are grouped into groups of three separated by thin spaces to make reading it easier. This is customary in the internationally used SI system (we will be looking at this system in detail in Chapter 3). In this book such grouping of digits will generally be used for numbers with six or more digits.

Solution

The easiest way to calculate this is to count the number of wells in each row (12) and column (8) and then multiply them:

$8 \times 12 = 96$

To calculate how many 96-well plates will be required for 960 047 tests, we need to divide 960 047 by 96 $(960\,047 \div 96)$ which gives us 10000 and a **remainder** of 47. This means we are able to fill 10000 plates to test 960 000 samples but then a further plate is required for the remaining 47 samples. In the last plate, 49 wells will remain empty $(96 - 47 = 49)$. In total, 10001 plates are needed. If you were to perform this calculation using a calculator, your answer would be 10000.48958, which is a decimal number.

1.1.3 *Estimation*

There are many situations in our everyday lives where we need to make an estimate instead of obtaining a precise answer. The problem below gives a good example of where we use **estimation**, which means we do not attempt to find the precise number but make a calculated guess that is near to the right answer.

Worked example 1.1.3

A biologist wants to conduct a study using bean plants and needs to decide how many plants to buy. They have 10 rows and each row is 72 cm in length. If the plants need to be spaced 7 cm apart in the row, how many plants should the biologist purchase?

Solution

The first step in solving this problem is to estimate how many plants can be placed in each row. If 72 cm is rounded down to 70 cm for the length of the row, then we can say that approximately $70/7 = 10$ plants can be placed in each row.
As there are 10 rows, then $10 \times 10 = 100$ plants are required.

SELF-ASSESSMENT

1.1.1 Soil samples are prepared for drying in an oven so that the moisture content can be measured by comparing the weight of the soil before and after drying. It takes a biology student 20 minutes to prepare and weigh a batch of nine samples. After spending 3 hours preparing samples, the student places them in the oven together with 6 samples that had been prepared the previous day. How many samples will there be in the oven?

1.1.2 A laboratory uses 4 vials per week of an enzyme for 52 weeks except for 5 weeks when some of the staff are on holiday and only 3 vials per week are required. How many vials does the laboratory use in total during all 52 weeks?

1.1.3 A student planning their research project has some samples that will be analysed using a spectrophotometer. The student needs to book the equipment, so they must estimate how long to make the booking for. It takes 1 minute 29 seconds for them to take readings for each sample and 20 seconds to change the sample. Estimate the length of time (in minutes) for which the student needs to book the spectrophotometer to carry out measurements for 30 samples.

MyMathLabGlobal

1.1.1 A lab needs to run toxicology tests using multi-well plates that hold 96 samples each. How many multi-well plates will the lab need for testing 289 150 samples?

1.1.2 A lab needs to order multi-well plates for conducting tests on 688 540 samples. Each multi-well plate will hold 96 samples. The supplier provides the plates in packs of 10. Calculate how many packs the lab will need to order.

1.1.3 An assay to obtain a standard curve will be conducted in triplicate and there will be nine standards with different concentrations used. How many test tubes will be needed for this assay?

1.1.4 How much buffer do you add to 10 μL of enzyme and 50 μL of substrate to obtain an enzymatic reaction mix with the total volume of 1200 μL?

1.1.5 A mixture of three different solvents (chloroform, ether and acetone) is prepared. A volume of 225 mL of chloroform is placed in a beaker, together with 373 mL of ether. How much acetone must be added for the final volume of the solution to be 800 mL?

1.1.6 An experimental subject in a pharmacological study must be given a dose of drug that is 8 mg for every kilogram of their body weight. Calculate the dose of drug for subjects with the following weights:

(a) 56 kg

(b) 75 kg

(c) 86 kg

1.1.7 You need to analyse 103 samples by electrophoresis using gels with 12 wells. In addition to your samples, you have to include molecular mass markers that will occupy one lane in each gel. How many gels do you need to run in total?

1.1.8 An enzyme assay uses 4 μL of enzyme solution. How many assays can you perform with the total of 720 μL of the enzyme solution, assuming no losses for pipetting?

1.1.9 A laboratory uses 7 bottles of distilled water every week of the year except for 9 weeks during the summer when its usage of distilled water is reduced to 5 bottles a week. How many bottles a year does the lab use?

1.1.10 A test tube rack can hold 24 test tubes. How many racks do you need to store 165 test tubes?

1.2 Indices, BODMAS and use of equations

1.2.1 *Indices*

Sometimes there are situations in which a number is multiplied by itself, e.g. 2×2. Another way of representing this would be as 2^2 which we commonly say is 2 squared or 2 raised to the power of 2. A similar example is $2 \times 2 \times 2$ which can be presented as 2^3, 2 raised to the power of 3, or 2 cubed. If we were to generalise, then this could be written as:

$$a^n$$

where a is the **base** and n is the **index** or **power**. The index represents the number of times that a should be multiplied by itself. The index is also sometimes referred to as the **exponent** or **order**. So a^n, where $n = 4$, would be written as:

$$a^4$$

which is the same as $a \times a \times a \times a$.
(Note that the plural of 'index' is **'indices'**.)
Any base raised to the power of 1 is equal to the base:

$$a^1 = a$$

For example: $7^1 = 7$

Laws of indices If numbers containing different bases are to be added, subtracted, multiplied or divided, their values must be calculated separately before calculating the sum, difference, **product** or **quotient**, respectively. This is illustrated in the next worked example.

Worked example 1.2.1

Evaluate:

(a) $2^3 + 4^2$
(b) $3^3 - 2^4$
(c) $2^3 \times 5^1$
(d) $2^6 \div 4^2$

Solution

(a) We need to calculate the values of 2^3 and 4^2 before carrying out the addition as the bases are different.
$2^3 = 2 \times 2 \times 2 = 8$
$4^2 = 4 \times 4 = 16$
So $2^3 + 4^2 = 8 + 16 = 24$

(b) We need to calculate the values of 3^3 and 2^4 before carrying out the subtraction as the bases are different.
$3^3 = 27$
$2^4 = 16$
$3^3 - 2^4 = 27 - 16 = 11$

(c) We need to calculate the values of 2^3 and 5^1 before carrying out the multiplication as the bases are different.
$2^3 = 8$
$5^1 = 5$
$2^3 \times 5^1 = 8 \times 5 = 40$

(d) We need to calculate the values of 2^6 and 4^2 before carrying out the division as the bases are different.
$2^6 = 64$
$4^2 = 16$
$2^6 \div 4^2 = 64 \div 16 = 4$

However, when calculations involve numbers with the same base, we can apply laws of indices.

First law of indices To multiply numbers that contain the same base, we add the indices:

$$a^m \times a^n = a^{m+n}$$

Worked example 1.2.2

Evaluate $2^3 \times 2^2$.

Solution

$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$

We can see that this is indeed the case when we write each term fully:

$2^3 = 2 \times 2 \times 2$
$2^2 = 2 \times 2$

So:

$2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

Second law of indices To divide numbers that contain the same base, we subtract the indices:

$$\frac{a^m}{a^n} = a^{m-n}$$

Worked example 1.2.3

Evaluate $\frac{2^3}{2^2}$.

Solution

$$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$$

We can show that this is the case when we write each term fully:

$$\frac{2^3}{2^2} = \frac{2 \times 2 \times 2}{2 \times 2} = 2$$

Third law of indices When we have a number raised to a power that is raised to a further power, we multiply the powers:

$$(a^m)^n = a^{m \times n}$$

Worked example 1.2.4

Evaluate $(2^2)^3$.

Solution

$(2^2)^3 = 2^{2 \times 3} = 2^6 = 64$

We can see that this is the case when we write this expression fully:

$(2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$

In the same way as there are positive and negative integers, there are both positive and negative indices. So far we have only considered examples where the index is positive. When it is zero, the fourth law of indices applies and when it is negative, the fifth law applies.

Fourth law of indices Any number raised to the power of 0 is equal to 1:

$$a^0 = 1$$

Worked example 1.2.5

Show that $2^0 = 1$.

Solution

We could write:

$2^0 = 2^{n-n}$

where n is any integer (because $n - n = 0$).

Using the second law of indices, we can express the right-hand side of the equation as:

$$2^{n-n} = \frac{2^n}{2^n}$$

This is equal to 1 as any number divided by itself is equal to 1.

So we have shown that $2^0 = 1$.

Fifth law of indices A number raised to a negative power is equal to 1 divided by this number raised to the positive power with the same absolute value:

$$a^{-m} = \frac{1}{a^m}$$

where $m > 0$.

Worked example 1.2.6

Evaluate 2^{-3}.

Solution

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

We can show that this is the case when we write:

$2^{-3} = 2^{0-3}$

Applying the second law of indices we have:

$$2^{0-3} = \frac{2^0}{2^3}$$

Since $2^0 = 1$, then:

$$\frac{2^0}{2^3} = \frac{1}{2^3}$$

So we have shown that:

$$2^{-3} = \frac{1}{2^3}$$

Sixth law of indices This law refers to fractional powers called roots.

$$a^{1/m} = \sqrt[m]{a}$$

For example:

$a^{1/2} = \sqrt{a}$ (square root)

$a^{1/3} = \sqrt[3]{a}$ (cube root)

Worked example 1.2.7

Evaluate $4^{1/2}$ and $8^{1/3}$.

Solution

$4^{1/2} = \sqrt{4}$

Square root of 4 has two values: 2 and −2, because both numbers squared give 4:

$2^2 = 4$ and $(-2)^2 = -2 \times (-2) = 4$

$8^{1/3} = \sqrt[3]{8} = 2$

The cube root of 8 has only one value as:

$2^3 = 8$ and $(-2)^3 = -2 \times (-2) \times (-2) = -8$

1.2.2 *BODMAS*

When a complex calculation has several steps, it is important to give priority to the parts of the calculation that need to be completed first, otherwise an incorrect answer may be produced. For example, let us calculate the value of the following expression:

$3 \times 5 - 1$

If the multiplication is performed first, then this would give $15 - 1 = 14$.
However, if the subtraction is (*incorrectly*) performed first this would give $3 \times 4 = 12$.
In maths, there is an established protocol for the sequence in which operations are performed in calculations. This is usually abbreviated as **BODMAS** which stands for:

B	**B**rackets first
O	**O**rder refers to powers
DM	**D**ivision and **M**ultiplication
AS	**A**ddition and **S**ubtraction